II-5. OPTIMUM MULTIPOLE QUARTER-WAVE TEM FILTERS

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Most microwave TEM filters use stub loading of a transmission line or coupled resonant elements to achieve the desired bandpass or bandstop characteristics. In many cases the stubs or resonant elements are of equal line length, and are spaced equally along a transmission line at quarter-wave intervals. The spacing between filter elements in these designs provides isolation from interaction with the fields of adjacent elements but otherwise does not contribute to the filtering characteristics. A theory is presented herein which includes the quarter-wave transmission line spacers, called "unit elements," in an exact design of maximally flat and equal ripple bandpass or bandstop response filters. The theory is applicable to all microwave filter forms consisting entirely of a cascade of quarter-wave lines, quarter-wave stubs, and coupled quarter-wave lines. Each line length element characterized in the theory may be used to create a complex plane pole to augment the filter skirt response. Thus, the new theory is termed "optimum multipole."

Modern network theory (References 1 through 5) can be applied to microwave filters and networks by using a frequency transformation,

$$S = j \tan \frac{\pi\omega}{2\omega_o} \tag{1}$$

introduced by Richards (Reference 6) in 1948. This transform relates the real microwave frequency parameter, ω, to an imaginary parameter, $S = j\Omega$, which can be used in a prototype design as the frequency of a corresponding lumped element filter. The correspondence between lumped element L's and C's and distributed shorted and opened lines (with Z_{oL} and Y_{oC} characteristic immittances) is given in Table I. Thus a lumped element high-pass prototype filter, having the transmission characteristic shown in Figure 1(a), would exhibit the corresponding bandpass characteristic of Figure 1(b) when L's and C's in the lumped element design are replaced by Z_{oL}'s and Y_{oC}'s, respectively, in the distributed microwave filter. However, the "unit element" transmission line spacer, which has no lumped element counterpart, must be characterized and incorporated into modern network approximation and synthesis procedures if it is to contribute to the filter response characteristic.

Two-port wave-cascading matrices, which relate input reflected and incident waves to output incident and reflected waves, respectively, are given in Table II as functions of S for high-pass ladder

Table I. Correspondence Between Lumped and Distributed Reactances

$Z = j\omega L$	$Z = \left(j \tan \frac{\pi\omega}{2\omega_o}\right) Z_{oL}$
$Y = j\omega C$	$Y = \left(j \tan \frac{\pi\omega}{2\omega_o}\right) Y_{oC}$

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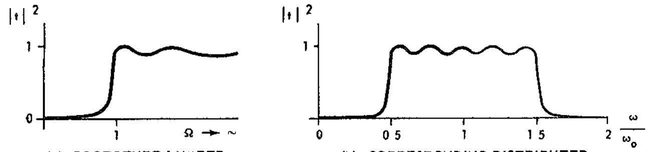
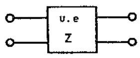
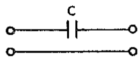
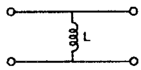


(a) PROTOTYPE LUMPED ELEMENT HIGH-PASS

(b) CORRESPONDING DISTRIBUTED ELEMENT BAND-PASS

Figure 1. Equal Pass Band Ripple Transfer Response

Table II. Wave Cascade Matrices for High-Pass Distributed Elements

SCHEMATIC	R-MATRIX
	$\frac{1}{\sqrt{1-S^2}}\left[(Z\tilde{A} + Z^{-1}A)S + I\right]$
	$\frac{1}{CS}(CSI + A)$
	$\frac{1}{LS}(LSI + \tilde{A})$
$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$A = \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$ $\quad \tilde{A} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

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"distributed L's and C's" (shorted and opened transmission lines) and for the unit element spacer. A cascade of these high-pass elements gives rise to an overall power ratio of reflection to transmission into the load given by

$$\frac{|\rho|^2}{|t|^2} = \left(\frac{S_c^{\,2}}{S^2}\right)^m \left(\frac{1 - S_c^{\,2}}{1 - S^2}\right)^n P_{m+n}\left(\frac{S^2}{S_c^{\,2}}\right) \tag{2}$$

where m is the number of L-C ladder elements, n is the number of unit elements, S_c is the prototype band-edge cutoff frequency, and P_{m+n} is an m+n order polynomial. The high-pass filter approximation problem is that of establishing proper coefficients of the polynomial to give a maximally flat (Butterworth) or equal ripple (Tschebyscheff) response in the pass band for the power transmission coefficient,

$$|t|^2 = \frac{1}{1 + |\rho|^2/|t|^2} \ . \tag{3}$$

The Butterworth approximation is given when the polynomial reduces to a unit constant, whence

$$\text{BUTTERWORTH:} \quad \frac{|\rho|^2}{|t|^2} = \left(\frac{S_c^{\,2}}{S^2}\right)^m \left(\frac{1 - S_c^{\,2}}{1 - S^2}\right)^n \ . \tag{4}$$

The Tschebyscheff approximation can be developed from consideration of a constant amplitude rational fraction all-pass function and is given by

$$\text{TSCHEBYSCHEFF:}\quad \frac{|\rho|^2}{|t|^2} = \epsilon^2 \left[T_m\left(\frac{S_c}{S}\right) T_n\left(\frac{\sqrt{1 - S_c^2}}{\sqrt{1 - S^2}}\right) - U_m\left(\frac{S_c}{S}\right) U_n\left(\frac{\sqrt{1 - S_c^2}}{\sqrt{1 - S^2}}\right)\right]^2 \qquad (5)$$

where $T_m(X) = \cos(m \text{ arc cos } X)$ and $U_m(X) = \sin(m \text{ arc cos } X)$ are un-normalized m^{th} degree Tschebyscheff polynomials of the first and second kinds, respectively, and ϵ^2 is the ripple power factor.

The equivalent low-pass prototype approximations are obtained from Equations (4) and (5) by substituting S^{-1} and S_c^{-1} for S and S_c. The resulting approximating functions may be used to synthesize low-pass prototypes of distributed bandstop TEM filters consisting of m L-C ladder distributed elements and n unit elements.

A seven-section equal-ripple bandpass distributed element filter (high-pass prototype, Figure 1) of 3:1 bandwidth was synthesized using the approximating function of Equation (5). The filter was chosen to consist of two unit elements and five L-C elements and was designed to have 0.1-db ripple in the pass band. The normalized cutoff frequency for a 3:1 band is $S_c = j$, and 0.1-db ripple corresponds to $\epsilon^2 = 0.0233$. From Equations (5) and (3):

$$|t|^2 = \frac{1}{1 + 0.0233\left[T_5\left(\frac{j}{S}\right) T_2\left(\frac{\sqrt{2}}{\sqrt{1 - S^2}}\right) - U_5\left(\frac{j}{S}\right) U_2\left(\frac{\sqrt{2}}{\sqrt{1 - S^2}}\right)\right]^2} \qquad (6)$$

which, after inserting the Tschebyscheff polynomials, can be solved for $|\rho|^2$ by again using Equation (3):

$$|\rho|^2 = \frac{\left[0.98S^6 + 8.15S^4 + 17.64S^2 + 10.78\right]^2}{-S^{14} + 2.96S^{12} + 14.96S^{10} + 100.96S^8 + 308.69S^6 + 487.06S^4 + 380.46S^2 + 116.23}\,. \qquad (7)$$

The complex plane roots of the denominator are obtained by use of a computer, left-half plane poles are associated with ρ, and the terminated filter input impedance is found from $Z_{in} = (1 + \rho)/(1 - \rho)$. Richards' Theorem and pole-removal techniques (Reference 1) can now be applied to Z_{in} to obtain the element values of the high-pass prototype network configuration shown in Figure 2. Kuroda's identities (Reference 1) are applied to this network to generate the configuration of Figure 3 which, although it contains two redundant elements, has element values more easily realizable. This network is realized in a circular cross section center conductor, shown in Figure 4, which is mounted between ground planes in TEM mode. The two series distributed "capacitors" are incorporated internal to the center conductor.

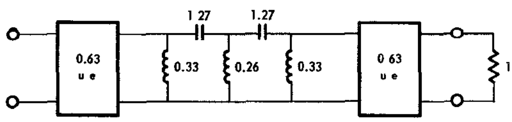

Figure 2. Seven-Section High-Pass Prototype Filter

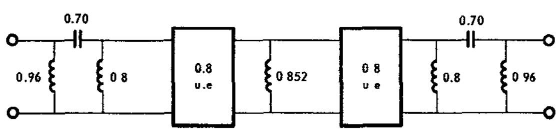

Figure 3. Application of Kuroda's Identities to Obtain Convenient Element Values

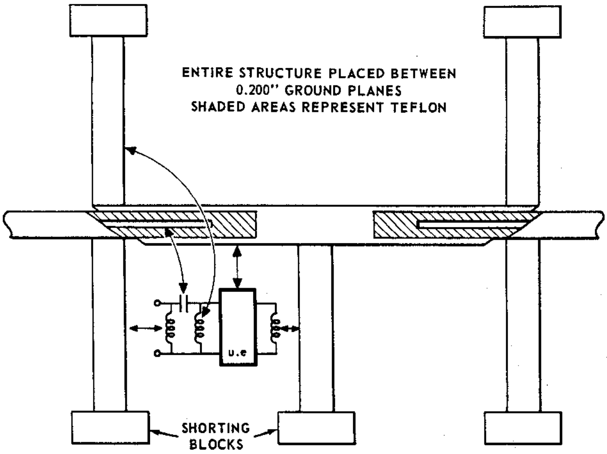

Figure 4. Detailed View of an Experimental Seven-Section Optimum Multipole Filter

An experimental model of this design was fabricated as a verification of the "optimum multipole" theory. The center pass band frequency of the model, shown in Figure 5, was 2.175 gc. The experimental response of the filter was found to be nearly identical to the theoretical characteristics as shown by the curves of Figure 6. Note especially the measured input VSWR which exhibits seven ripples and demonstrates that the two unit elements are contributing to the filter characteristic.

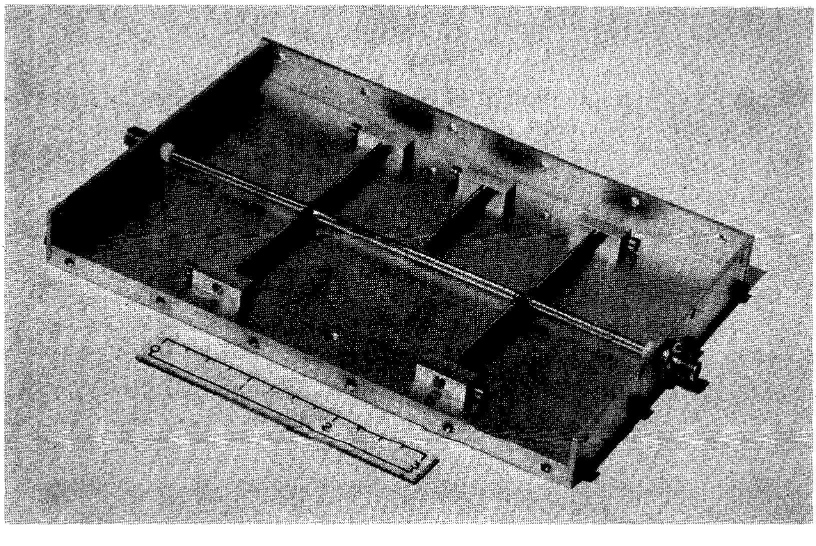

Figure 5. Experimental Seven-Section Optimum Multipole Filter

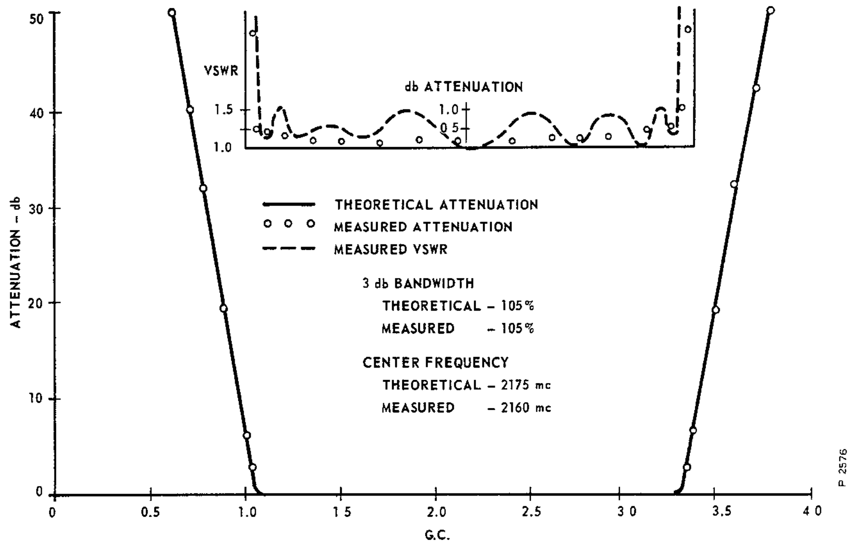

Figure 6. Theoretical and Experimental Response Characteristics of a Seven-Section Optimum Multipole Filter

The approximation and synthesis techniques developed for optimum multipole filter design are applicable to a broad class of TEM filter forms employing quarter-wave lines. Every quarter-wave element, including unit elements, can be used to obtain optimum response characteristics in a minimum size. The stub (L-C) elements are always superior to unit elements in filtering capability; however, the unit elements are quite effective in bandpass (high-pass prototype) designs of one octave or less, or in wide bandstop (low-pass prototype) designs.

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